**Specialized Averages: The Geometric & Harmonic Means**

**The Geometric Mean:** The geometric mean (G) of n non-negative numerical values is the nth root of the product of the n values. The Geometric mean (G) of n positive numerical values x(i) is: G = [product of all x(i )] 1/n.

If some values are very large in magnitude and others are small, then the geometric mean is a better representative of the data than the simple average. In a” geometric series", the most meaningful average is the geometric mean (G). The arithmetic mean is very biased toward the larger numbers in the series.

**An Application:** Suppose sales of a certain item increase to 110% in the first year and to 150% of that in the second year. For simplicity, assume you sold 100 items initially. Then the number sold in the first year is 110 and the number sold in the second is 150% x 110 = 165. The arithmetic average of 110% and 150% is 130% so that we would incorrectly estimate that the number sold in the first year is 130 and the number in the second year is 169. The geometric mean of 110% and 150% is G = (1.65)1/2 so that we would correctly estimate that we would sell 100 (G)2 = 165 items in the second year.

**The Harmonic Mean:** The harmonic mean (H) is another specialized average, which is useful in averaging variables expressed as rate per unit of time, such as mileage per hour, number of units produced per day. The harmonic mean (H) of n positive numerical values x(i) is: H = n/[S (1/x(i)].

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**An Application:** Suppose 4 machines in a machine shop are used to produce the same part. However, each of the four machines takes 2.5, 2.0, 1.5, and 6.0 minutes to make one part, respectively. What is the average rate of speed?

The harmonic means is: H = 4/[(1/2.5) + (1/2.0) + 1/(1.5) + (1/6.0)] = 2.31 minutes.

If all machines working for one hour, how many parts will be produced? Since four machines running for one hour represent 240 minutes of operating time, then: 240 / 2.31 = 104 parts will be produced.

**The Order Among the Three Means:** If all the three means exist, then the Arithmetic Mean is never less than the other two, moreover, the Harmonic Mean is never larger than the other two.

You might like to use:

**http://home.ubalt.edu/ntsbarsh/Businessstat/otherapplets/ThreeMeans.htm**

Use this use JavaScript in performing some numerical experimentation for validating the above assertions for a deeper understanding.

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